Correspondence

The Characteristic Impedance of Square Coaxial Line

In a recent communication, Green [1] has given approximate results for the characteristic impedance of a coaxial line with square section inner and outer conductors. It is the purpose of this communication to point out that Bowman [2] has given an exact solution in terms of elliptic integrals, which is by no means computationally tedious. It is, in Green's notation

$$Z_0 = 5\pi v \times 10^{-8} K'(k)/K(k) \qquad (1)$$

where the modulus $k=(\lambda'-\lambda)^2/(\lambda'+\lambda)^2$ and λ and $\lambda'=(1-\lambda^2)^{1/2}$ are given implicitly in terms of the line dimensions by

$$K'(\lambda)/K(\lambda) = (b+a)/(b-a). \qquad (2)$$

The calculations may be carried out using tables of elliptic integrals, or by introducing the modular constant

$$q = \exp(-\pi K'/K)$$

and using the rapidly convergent theta function series [3]

$$\sqrt{k'} = \frac{1-2q+2q^4-2q^9\cdots}{1+2q+2q^4+2q^9\cdots}$$

and

$$q = \epsilon + 2\epsilon^5 + 15\epsilon^9 \cdots$$

where

$$2\epsilon = \frac{1-\sqrt{k'}}{1+\sqrt{k'}}$$

to find q from k' and vice versa. Tables of K'/K against values of k^2 have also been published [4].

Eqs. (1) and (2) may equally well be used to determine the line dimensions for a given characteristic impedance. For example, taking $v=2.9979\times10^8$ m/sec, we find that for $Z_0=50\ \Omega$, these are given by $b/a=2.5076$, which is 0.14 per cent below the value in Green's Table II.

When b/a is a simple fraction, the modulus λ in (2) can be determined algebraically [3] from the theory of modular transformations, often with some simplification of the work. Thus, in the case $b/a=3$ discussed by Green, we have $K'/K=2$ and $\lambda=(\sqrt{2}-1)^2$, and the characteristic impedance is 60.6106 Ω, which is 0.14 per cent above the value in Green's Table I.

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References

[1] H. E. Green, "The characteristic impedance of the square coaxial line," IEEE Trans. on Microwave Theory and Techniques, vol. MTT-11, pp. 554–555; November, 1963.
[2] F. Bowman, "Introduction to Elliptic Functions," English Universities Press, London, ch. 10; 1953.
[3] E. T. Whittaker and G. N. Watson, "A Course of Modern Analysis," Cambridge University Press, Cambridge, England, ch. 21 and 22; 1927.
[4] F. Oberhettinger and W. Magnus, "Anwendung der Elliptischen Funktionen in Physik und Technik," Springer-Verlag, Berlin, Germany; 1949.

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A Varactor Tuned UHF Coaxial Filter

The purpose of the following communication is to present a method for designing a varactor-tuned UHF coaxial filter.[1] The method includes the means for determining the varactor diode characteristics necessary for a given tuning range and bandwidth.

The basic idea of the filter consists of an LC circuit where L is realized by a short-circuited coaxial line, and C is realized by the sum of the junction capacitance and cartridge capacitance of the varactor diode. It is desirable to use a diode in which the cartridge capacitance is small compared to the junction capacitance in order to maximize the contribution of the variable junction capacitance.

The geometry of a practical half-wave filter is shown in Fig. 1. Because of the balanced geometry of the half-wave resonator, an antinode of current exists at the diode position which will minimize the signal voltage drop across the diode. The button capacitor at one end of the resonator provides an RF short and dc isolation for the diode biasing potential. The diode biasing potential is applied between the center terminal of the button capacitor and the outer coaxial conductor. Furthermore, the use of diode holders (adapters) has the important advantage that they can be soldered to the two parts of the inner conductor while demounted from the diode cartridge.

The equivalent circuit of the resonator is shown in Fig. 2. The diode impedance is closely given by

$$Z_d = r_s + 1/j\omega C_t \qquad (1)$$

when $\omega^2 R_s C_c C_j \ll 1$ and $(\omega R_s C_c C_j/C_t)^2 \ll 1$, where

$$c_t = C_c + C_j \qquad (2)$$

$$r_s = R_s(C_j/C_t)^2. \qquad (3)$$

For the unloaded resonator, at resonance ($\omega=\omega_0$) the magnitude of the capacitive reactance is equal to that of the total inductance; hence, at resonance,

$$\frac{1}{2C_t} = \omega_0 R_0 \tan\frac{\omega_0 l}{v} \equiv \omega_0 Z \qquad (4)$$

where R_0, l, and v are the characteristic impedance of the coaxial line comprising the resonator, the half length of the resonator center conductor, and the velocity of propagation within the resonator, respectively.

In regard to the length l, a suitable value is one half of the physical length of the inside of the resonator. Using such a value imposes a slight approximation because the ceramic diode cartridge makes the overall center conductor electrical length slightly longer than the geometric length. Furthermore, the resonator length reduction due to an increase in the dielectric constant is partially cancelled due to the curvature of the electric field lines at the middle of the resonator. Also, experiment has shown that it is an excellent approximation to consider the geometric half length of the inside of the resonator as the length l.

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[1] A. P. Benguerel, "Coaxial Filter Tuned with a Varactor Diode," M.Sc. thesis, Dept. of Electrical Engineering (ERL), University of Kansas, Lawrence, Kan.: 1962

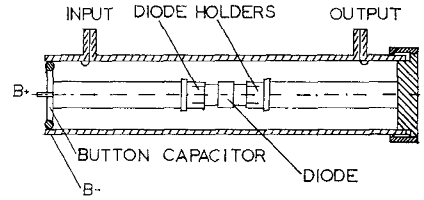

Fig. 1—The coaxial filter.

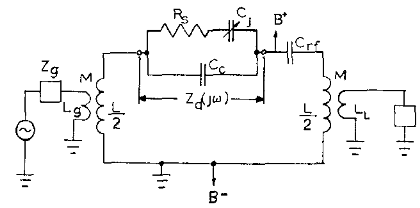

Fig. 2—The resonator equivalent circuit.

When the coupled reactive component is negligible compared to $\omega_0 L$, the reciprocal of the loaded Q can be expressed as

$$\frac{1}{Q_L} = \frac{r_s}{\omega_0 L} + \frac{2\omega_0^2 M^2 Z_0}{\omega_0 L(Z_0^2 + \omega_0^2 L_0)} \qquad (5)$$

where the first and second terms are the reciprocals of the unloaded resonator Q and the external Q component, Q_u and Q_e, respectively. The insertion ratio of the filter is given by

$$\frac{P_{\text{out}}}{P_{\text{in}}} = \left(\frac{r_s Q_u}{|Z_t|\,Q_e}\right)^2 \qquad (6)$$

where

$$Z_t = r_s\left[1 + \frac{Q_u}{Q_e}\left(1 - j\frac{\omega_0 L_0}{Z_0}\right)\right]. \qquad (7)$$

From (4), two design curves can be derived. The first one, Fig. 3, gives the values of ωZ as a function of f with l as a parameter. The second one, Fig. 4, gives $\omega Z = \frac{1}{2}C_t$ as a function of the varactor bias voltage V_b with the zero bias junction capacitance C_{j0} as a parameter. The latter curve is based upon the assumption that the junction capacitance is accurately given by

$$C_j(V_b) = C_{j0}\left(1 - \frac{V_b}{V_0}\right)^{-1/k} \qquad (8)$$

where C_{j0}, V_0, and k are constants. Numerical values of these constants for a given diode are usually contained in the manufacturer's description of the diode; if their values are not given, they can be derived from a plot of $C_j(V_b)$ vs V_b. Similarly, by virtue of (3), a third design curve, Fig. 5,

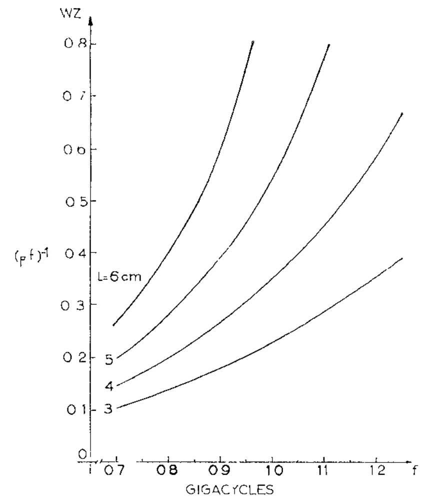

Fig. 3—Design curve ωZ vs f. Resonator line characteristic impedance is 50 Ω.

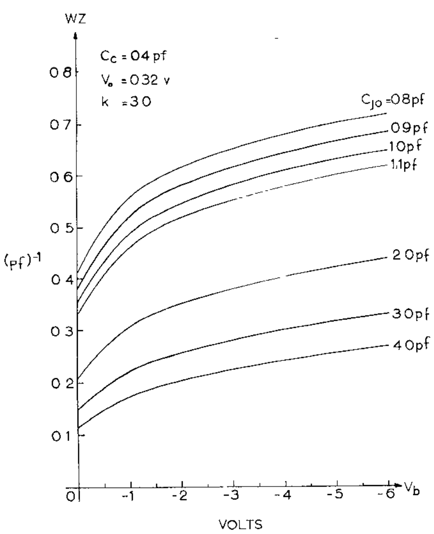

Fig. 4—Design curve ωZ vs V_b.

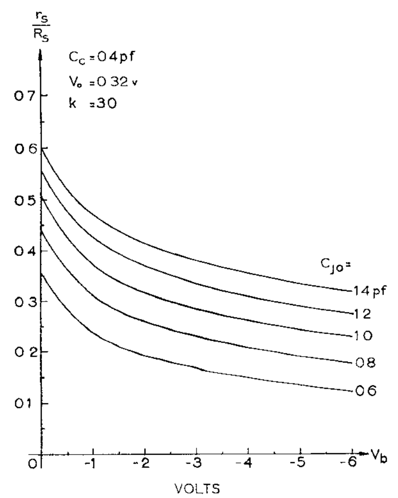

Fig. 5—Design curve r_s/R_s vs V_b.

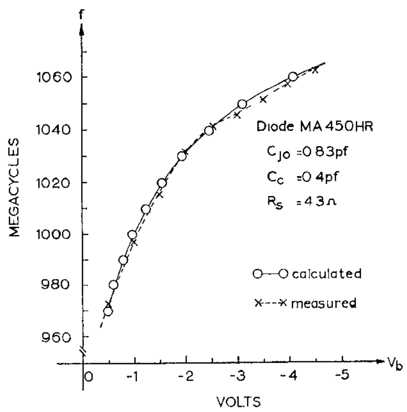

Fig. 6—Experimental results; tuning curve.

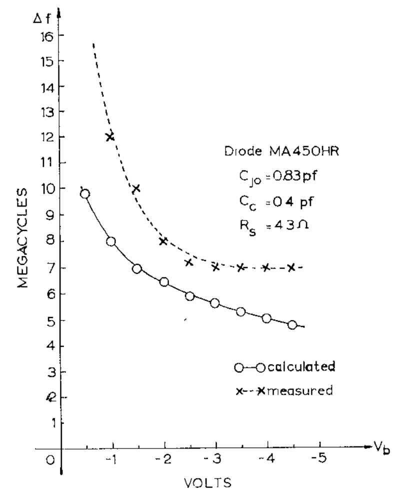

Fig. 7—Experimental results; bandwidth.

can be derived. The third curve is a plot of the quantity r_s/R_s as a function of the bias voltage V_b with the zero bias junction capacity C_{j0} as a parameter. The curve shown in Figs. 3 through 5 have been constructed for one series of diodes, the MICROWAVE ASSOCIATES MA 450 series which consists of eight diode models, all having the same C_c, V_0, and k.

To use the design curves, the procedure is as follows. Given a certain frequency range to cover, *e.g.*, f_1 to f_2, there are five diode and two resonator parameters to be selected. The diode parameters are C_{j0}, C_c, k, V_0 and V_m, the diode reverse bias breakdown voltage; the resonator parameters are R_0 and l.

A convenient design sequence is as follows:

1) Choose a particular series of diodes; this specifies C_c, V_0, and k.
2) Choose a value for the resonator half length l and characteristic impedance R_0.
3) Refer to the design curve ωZ vs V_b (Fig. 3) and read the two values of ωZ: $(\omega Z)_1$ and $(\omega Z)_2$ for the chosen l at the two frequencies f_1 and f_2.
4) Refer to the design curve ωZ vs V_b (Fig. 4) along the lines $(\omega Z)_1$ and $(\omega Z)_2$ and determine the C_{j0} values whose curves for a fixed value of C_{j0} interest both of the curves $\omega Z=(\omega Z)_1$ and $(\omega Z)_2$.
5) Compare the C_{j0} values with the values of available diodes. Failure to obtain an applicable value of C_{j0} will necessitate a second try by changing l or C_c or both.
6) Determine the resonator bandwidth Δf from the r_s/R_s vs V_b curve (Fig. 5) and the relation

$$\Delta f = \frac{f_0}{Q_u} = 2\pi f_0{}^2 r_s C_t. \tag{9}$$

Figs. 6 and 7 are typical results obtained with the above procedure. The desired frequency range was 970 Mc to 1060 Mc; also, the resonator half length l was 5 cm.

The predicted curves are for the unloaded resonator while the measured curves are for the complete filter (resonator and coupled loads). The generator and load impedance were each 50 Ω. The closeness of the measured and predicted tuning curves indicates that the coupling coefficients between the resonator and the 50 Ω coaxial loads were very small. The influence of coupling can be estimated by comparing the two bandwidth curves; the difference between the curves can be interpreted through (5).

It has been shown that the design of a UHF varactor-tuned coaxial filter can be conveniently accomplished through the use of design curves which characterize the diode. The design sequence is useful for determining the diode specifications necessary for a given tuning range and bandwidth.

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The Equivalent Circuit of a Plasma

Several recent papers, *e.g.*, Kaufmann and Steier,[1] show how the plasma resonance can be used to achieve rejection and transmission filters.

The object of this communication is to derive the equivalent circuit of two typical configurations and to discuss the bandwidth of the resultant filter. The equivalent dielectric constant of plasma at radial frequency ω is known to be

$$\epsilon_p = \epsilon_0\left(1 - \frac{\omega_p{}^2}{\omega(\omega - j\nu)}\right)$$

where the symbols have the usual meaning. Consider a slab of uniform plasma of thickness d between two plates of contact area A;

Manuscript received March 2, 1964.

[1] I. Kaufman and W. H. Steier, "A plasma-column band-pass microwave filter," IRE Trans. on Microwave Theory and Techniques, vol. MTT-10, pp. 431–439; November, 1962.